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The instability of the spiral wave induced by the deformation of elastic excitable media

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Abstract
There are some similarities between the spiral wave in excitable media and in cardiac tissue. Much evidence shows that the appearance and instability of the spiral wave in cardiac tissue can be linked to one kind of heart disease. There are many models that can be used to investigate the formation and instability of the spiral wave. Cardiac tissue is excitable and elastic, and it is interesting to simulate the transition and instability of the spiral wave induced by media deformation. For simplicity, a class of the modified Fitzhugh–Nagumo (MFHN) model, which can generate a stable rotating spiral wave, meandering spiral wave and turbulence within appropriate parameter regions, will be used to simulate the instability of the spiral wave induced by the periodical deformation of media. In the two-dimensional case, the total acreage of elastic media is supposed to be invariable in the presence of deformation, and the problem is described with $L_x \times L_y = N \times \Delta x N \times \Delta y = L'_x L'_y = N \times \Delta x' N \times \Delta y'$. In our studies, elastic media are decentralized into $N \times N$ sites and the space of the adjacent sites is changed to simulate the deformation of elastic media. Based on the nonlinear dynamics theory, the deformation effect on media is simplified and simulated by perturbing the diffusion coefficients D_x and D_y with different periodical signals, but the perturbed diffusion coefficients are compensatory. The snapshots of our numerical results find that the spiral wave can coexist with the spiral turbulence, instability of the spiral wave and weak deformation of the spiral wave in different conditions. The ratio parameter ε and the frequency of deformation forcing play a deterministic role in inducing instability of the spiral wave. Extensive studies confirm that the instability of the spiral wave can be induced and developed only if an appropriate frequency for deformation is used. We analyze the power spectrum for the time series of the mean activator of four sampled sites which are selected symmetrically in different cases, such as the condition that the spiral wave coexists with the spiral turbulence, spiral wave without evident deformation, complete instability

of the spiral wave (turbulence) and weak deformation of the spiral wave. It is found that more new peaks appear in the power spectrum and the distribution of frequency becomes sparser when the spiral wave encounters instability.

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1. Introduction

Spiral waves are often observed in the reaction–diffusion system [1–6], such as the well-known Belousov–Zhabotinsky (BZ) reaction [5–7], electrical activity in cardiac tissue [9, 10] and CO oxidation on Pt(1 1 0) surface [3, 8, 11], and much attention has been paid to the mechanism of appearance and instability [12–15] of the spiral wave. There are some similarities between the spiral waves in the excitable media and those in the cardiac tissue. It is found that the instability of the spiral wave can induce spiral turbulence, and the breakup of the spiral wave in cardiac tissue can cause a rapid death of the heart, which is called ventricular fibrillation [16, 17]. Extensive investigations have been carried out so that some effective schemes [18, 19] can be used to prevent the appearance and breakup of the spiral wave in cardiac tissue. It does not mean that the topic of the spiral wave just focuses on removing the useless traveling wave and turbulence in cardiac tissue because the spiral wave is also a hot topic in pattern formation. Usually, the modified Fitzhugh–Nagumo equation (MFHN) [20, 21] is often used to describe the spiral wave and spiral turbulence in the excitable and/or oscillatory media in appropriate parameter regions, respectively.

To date, many schemes have been proposed to eliminate the spiral wave and turbulence in excitable and/or oscillatory media. These schemes include periodical forcing [22–25], feedback [26–30], parameter perturbation [31, 32], etc. On the other hand, the mechanism of the spiral wave breakup has been investigated extensively [33–37]. It is reported that external forcing can induce geometrical deformation in flexible media and thus patterns are generated [38]. In [39], Zhang *et al* reported that the mechanical force can cause breakup of the spiral wave in excitable media. As we know, cardiac tissue can be considered elastic tissue, and the periodical shrinkage and stretch can induce transition of a traveling wave in cardiac tissue. Therefore, it is more interesting to extend the previous results [39] in which they discussed the breakup of the spiral wave by imposing lateral mechanical force on the media (along the long direction or landscape orientation, along the x- or y-axis). In this paper, we investigate the instability of the spiral wave induced by the geometrical deformation of media along lengthways and transverse ways (along the x and y axes). Furthermore, we will discuss how to detect and identify the instability of the spiral wave by analyzing the distribution of frequency and mean frequency of a few sampled sites (activator). In the two-dimensional spatial space, the elastic media require that the total square area of the system should be supposed to be invariable during the course of system deformation. For example, the length size L_x and breadth L_y are replaced by the variables L'_x and L'_y, and it requires that the formula $L'_x \times L'_y = L_x \times L_y$ should be satisfied as well. In our numerical simulation tests, the elastic media are decentralized into $N \times N$ sites and the deformation effect on the transition of the spiral wave will be discussed with the nonlinear dynamics theory. It will be confirmed that the deformation effect can be simulated and described by changing the diffusion coefficients D_x and D_y with different functions (D_x increases but D_y decreases, vice versa), and it is different from the previous works which just changed the controllable diffusion coefficients D_x and D_y by the same step or function.

2. Model and geometrical deformation of media

2.1. A model of the modified Fitzhugh–Nagumo model

The modified Fitzhugh–Nagumo (MFHN) model [20, 21] is often used to describe the dynamics of the spiral wave in the excitable media. It confirmed that meandering spiral wave and spiral turbulence can also be observed in the MFHN model within appropriate parameters. In this paper, the MFHN model is used to describe the transition of the spiral wave in excitable media by selecting appropriate parameters, and the model is often given with

$$\begin{cases} \dfrac{\partial u}{\partial t} = \varepsilon^{-1} u(1-u)\left(u - \dfrac{v+b}{a}\right) + D\nabla^2 u \\ \dfrac{\partial v}{\partial t} = f(u) - v \end{cases} \tag{1}$$

$$f(u) = \begin{cases} 0 & 0 \leqslant u < 1/3 \\ 1 - 6.75u(u-1)^2 & 1/3 \leqslant u \leqslant 1 \\ 1 & 1 < u. \end{cases} \tag{2}$$

System (1) is an activator controller two-variable reaction–diffusion model, where variables u and v describe the activator and inhibitor respectively. D is the diffusion coefficient, a, b and ε are parameters. In the case of the two-dimensional space $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, the dynamics is investigated by varying ϵ and fixing $a = 0.84$ and $b = 0.07$. The dynamics of the system is decided by the decisive parameters b and ε; the parameter ε describes the ratio of the time scales of the fast activator u and the slow inhibitor variable v, and parameter b determines the excitation threshold. The three parameters are endowed with different physical meanings for different real systems; for example, the external controllable parameters (the crystal temperature, partial pressures of CO and oxygen) in the model of the CO oxidation on Pt(1 1 0) are mapped into the three parameters a, b and ε. It is found that the spiral wave is stable when $0.01 < \epsilon < 0.06$ at $a = 0.84$ and $b = 0.07$, while the spiral wave becomes unstable for $\varepsilon > 0.06$ and it finally steps into turbulence after $\epsilon > 0.07$. In this paper, time step size $h = 0.02$, sites $N \times N = 256 \times 256$, site space $\Delta x = \Delta y = 100/256$ and the diffusion coefficient $D = 1$ without a special statement. The numerical algorithm applied for the simulation is the Euler forward difference algorithm with the non-flux boundary conditions being used.

2.2. The problem of the geometrical deformation

As mentioned above, the spiral wave in cardiac tissue is harmful and the instability of the spiral wave in cardiac tissue is dangerous. There are many theoretical models to study the formation and instability of the spiral wave in excitable and oscillatory media, and we would use the famous MFHN model to study the instability of the spiral wave in excitable media when the deformation of the elastic media is considered. As we know, cardiac tissue is elastic and excitable; in the two-dimensional case, the property of elastic can be simplified as the size of the media is kept invariable. On the other hand, any electric shock or external forcing imposed on cardiac tissue often causes it to shrink or expand in rhythm. For simplicity, we suppose that the media are deformed periodically. Now we will use the nonlinear dynamics theory to describe and discuss these problems according to the assumptions mentioned above. In our studies, the size of the media is supposed to be $L_x \times L_y$ and the media are composed of $N \times N$ sites before deformation is imposed. The size of the media is described by $L'_x \times L'_y$

and the sites of the media are still $N \times N$. Therefore, the geometrical deformation of the elastic media can be described by

$$L_x \times L_y = N \times \Delta x N \times \Delta y = L'_x L'_y = N \times \Delta x' N \times \Delta y', \tag{3}$$

where the sites of the media are supposed to be invariable and the deformation just changes the space size of the adjacent sites. It is interesting to mark the new position of the sites with (x', y') and the original position with (x, y) though we cannot assure the correlation between the deformation degree and the external forcing exactly. Therefore, we will discuss the transition of the spiral wave in a theoretical and numerical way. Then the correlation of sites position can be described by

$$x'(t) = Af(t)x(t); \qquad y'(t) = y(t)/(Af(t)), \tag{4}$$

where $Af(t)$ describes the degree of geometrical deformation, and A is the maximal amplitude of deformation and $A > 0$. $0 \leqslant x = (i-1)\Delta x \leqslant L_x, 0 \leqslant y = (j-1)\Delta y \leqslant L_y, 0 \leqslant x' = (i-1)\Delta x' \leqslant L'_x, 0 \leqslant y' = (j-1)\Delta y' \leqslant L'_y, i, j = 1, 2, 3, \ldots, 256$. Now it is important to discuss and explain the transformation shown in equation (4),

$$L_x = L_y = L_0, \qquad \Delta x = \Delta y = L_0/N \tag{5}$$

$$L'_x = L_0 + L_0 Af(t) = L_0(1 + Af(t)), \qquad L'_y = L_0^2 \big/ L'_x = L_0/(1 + Af(t)) \tag{6}$$

$$\Delta x' = L'_x/N = (1 + Af(t))\Delta x, \qquad \Delta y' = L'_y/N = \Delta y/(1 + Af(t)). \tag{7}$$

According to equation (3), the number of sites is supposed to be invariable and each site (x, y) can be marked with (i, j) and $x = (i-1)\Delta x, y = (j-1)\Delta y$, $i, j = 1, 2, 3, \ldots, N, x' = (i-1)\Delta x', y' = (j-1)\Delta y'$. Clearly, the diffusion factor $D\nabla^2$ is sensitive to the change of space size of the adjacent sites and the problem can be defined by

$$D\frac{\partial^2}{\partial x'^2} = D\frac{\partial}{\partial x'}\left(\frac{\partial}{Af(t)\partial x}\right) = \frac{D}{A^2 f^2(t)}\frac{\partial^2}{\partial x^2} \tag{8}$$

$$D\frac{\partial^2}{\partial y'^2} = D\frac{\partial}{\partial y'}\left(\frac{Af(t)\partial}{\partial y}\right) = \frac{DA^2 f^2(t)}{1}\frac{\partial^2}{\partial y^2}. \tag{9}$$

According to equation (1), from a mathematical point of view, the parameter for the diffusion factor is changed as the deformation is considered. Equations (8) and (9) show that the diffusion coefficients are changed as $D'_x = DA^2 f^2(t) = D_x A^2 f^2(t)$ and $D'_y = D/(A^2 f^2(t)) = D_x/(A^2 f^2(t))$. In a practical way, it differs from the popular scheme by perturbing the diffusion coefficient with the time function arbitrarily, for example, $D'_x = f(t)D_x$ and $D'_y = f(t)D_y$. In this paper, equations (7)–(9) throw light on the dynamics and transition of the spiral wave defined with equation (1). In fact, the deformation degree is reasonable within an appropriate threshold for the elastic media. For simplicity, it is supposed to be the maximal amplitude of the deformation $A_{\max} = 0.4$ in this paper. As we know, the ratio parameter ε also plays an important role in the transition of the spiral wave, so it is necessary to discuss the problem with a different ratio parameter ε.

3. Numerical simulation results and discussions

In the following numerical simulation, appropriate initials are used to generate a stable rotating spiral wave in the media with a transient period of about 200 time units and then the geometrical

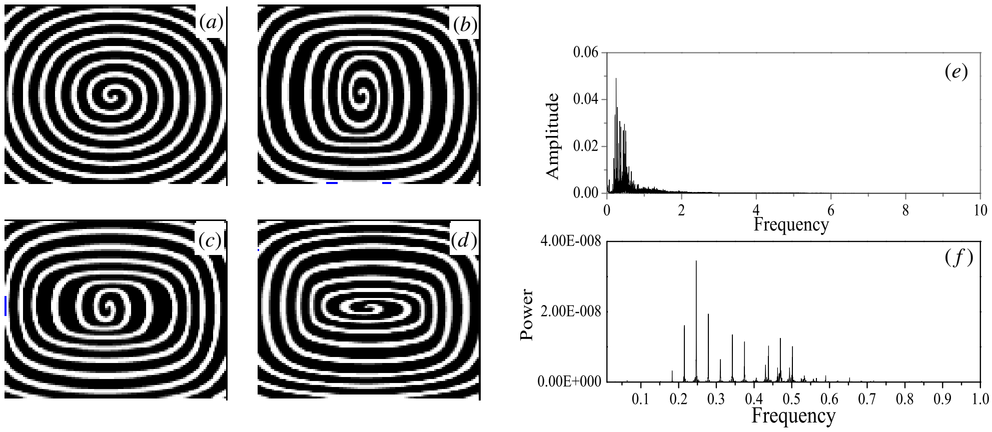

Figure 1. Evolution of the deformed spiral wave, $\varepsilon = 0.02$, $Af(t) = 0.4\cos 0.2t$, for $t = 0$ (*a*), $t = 1000$ (*b*), $t = 1400$ (*c*) and $t = 1800$ (*d*) time units, and the snapshots are shown in gray. Power spectrum for the time series of mean activator of the four sampled sites $u(88, 128)$, $u(128, 88)$, $u(128, 168)$, $u(168, 128)$.

deformation will be introduced into the spiral wave. Parameters $a = 0.84$, $b = 0.07$, sites number $N = 256$, time step $h = 0.02$ and site space $\Delta x = \Delta y = 100/256$. The elastic media responds to the external deformation synchronically as the external forcing is imposed on the whole media, no matter whether it is periodical or not. In this paper, we focus on the problem that the elastic media stretch and/or shrink in a periodical way due to the external periodical forcing on the media. Therefore, the problem is simplified as $Af(t) = A_{\max}\cos\omega t$, and the parameter ω is used to describe the angle frequency of the deformation. The snapshots of the spiral wave are observed and plotted to discuss the transition of the spiral wave induced by the external periodical forcing-induced deformation of the media. In our studies, we mainly discuss the transition of the spiral wave induced by the action of a different angle frequency ω and ratio parameter ε.

The snapshots of figure 1 confirm that the spiral wave is just compressed in one direction while another orthogonal direction is stretched. Furthermore, other angle frequencies $\omega = 0.05, 0.1, 0.3, 0.4$ are checked for $\varepsilon = 0.02$ and it is found that the spiral wave is shrinked or stretched alternatively. To estimate the distribution of frequency, the time series of four sampled sites $u(88, 128)$, $u(128, 88)$, $u(128, 168)$, $u(168, 128)$ are analyzed with an FFT scheme, and the power spectra are plotted in figures 1(*e*) and (*f*). Clearly, the four sampled sites lie in the media symmetrically along four different directions so that it can foretell the change of the traveling wave in the media. We will compare the distribution of frequency in figure 1 with other cases in the following to estimate whether the instability of the spiral wave is induced.

Then we investigate the case for $\varepsilon = 0.03$ and it just finds that the spiral wave is stretched or shrinked for $\omega = 0.05, 0.1, 0.2, 0.3, 0.4, 0.5$. Extensive numerical studies are given to investigate the problem for $\varepsilon = 0.04$ at different angle frequency ω. It is found that the spiral wave can coexist with the spiral turbulence at $\omega = 0.1$ with $\varepsilon = 0.04$ while other angle frequencies $\omega = 0.05, 0.2, 0.3, 0.4, 0.5$ just cause the spiral wave to become deformed without instability. Then the ratio parameter ε is increased to investigate the problem with other angle frequencies. It is found that the transition and evolution of the spiral wave is complex at $\varepsilon = 0.045$, and the relevant results are plotted in figure 2.

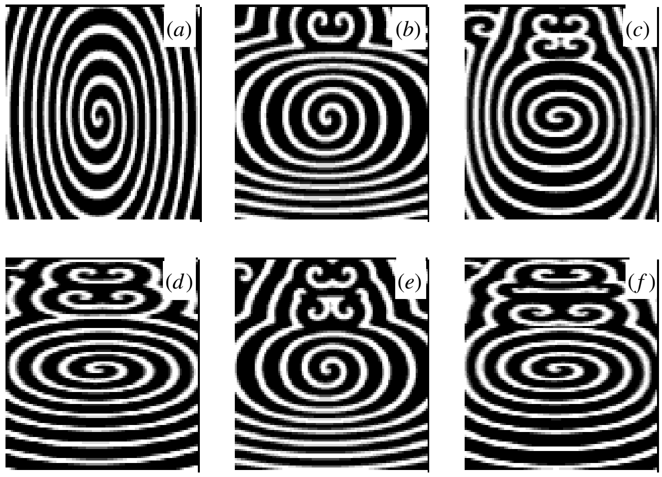

Figure 2. Evolution of the deformed spiral wave, $\varepsilon = 0.045$, $Af(t) = 0.4\cos 0.05t$, for $t = 200$ (*a*), $t = 400$ (*b*), $t = 600$ (*c*), $t = 1000$ (*d*), $t = 1400$ (*e*) and $t = 1800$ (*f*) time units, and the snapshots are shown in gray.

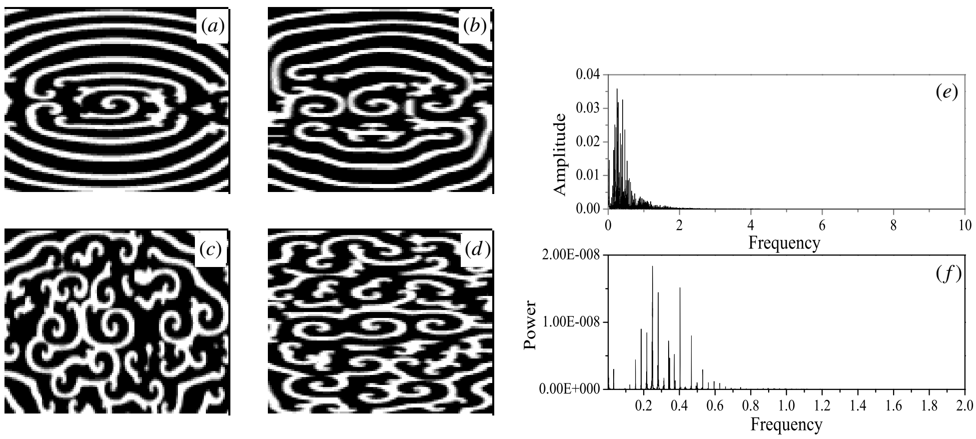

Figure 3. Evolution of the deformed spiral wave, $\varepsilon = 0.045$, $Af(t) = 0.4\cos 0.2t$, for $t = 200$ (*a*), $t = 600$ (*b*), $t = 1400$ (*c*) and $t = 1800$ (*d*) time units, and the snapshots are shown in gray. Power spectrum for the time series of mean activator of the four sampled sites $u(88, 128)$, $u(128, 88)$, $u(128, 168)$, $u(168, 128)$.

The snapshots of figure 2 show that the spiral wave can coexist with the spiral turbulence and the spiral wave is surrounded by the spiral turbulence at $\omega = 0.05$. Then other angle frequencies are checked and it is found that the spiral wave still can coexist with the spiral turbulence though the area of the spiral turbulence becomes bigger at $\omega = 0.08, 0.09, 0.1, 0.12$. Then the angle frequency is increased further and the results are plotted in figure 3.

The snapshots of figure 3 confirm that the spiral wave begins to lose stability from the center of the media and finally the whole media encounter complete instability. The center instability seems like the Doppler instability, which results from the meandering of the tip

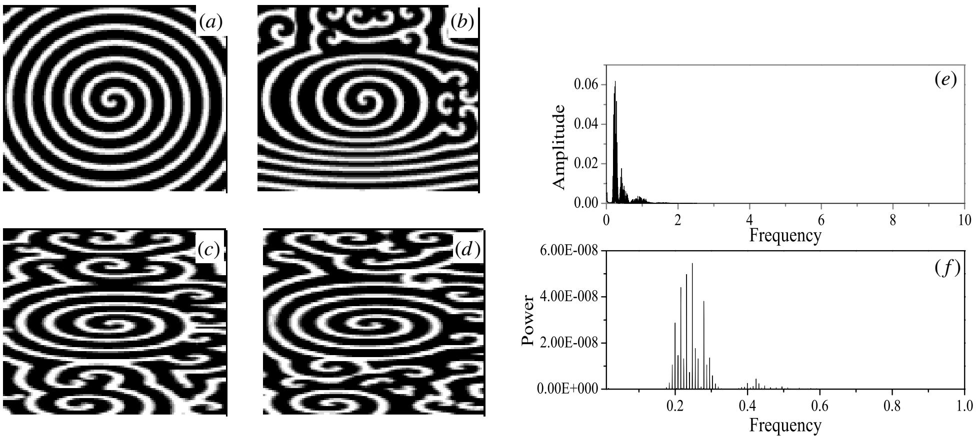

Figure 4. Evolution of the deformed spiral wave, $\varepsilon = 0.05$, $Af(t) = 0.4\cos 0.05t$, for $t = 200$ (*a*), $t = 1000$ (*b*), $t = 1400$ (*c*) and $t = 1600$ (*d*) time units, and the snapshots are shown in gray. Power spectrum for the time series of mean activator of the four sampled sites *u*(88, 128), *u*(128, 88), *u*(128, 168), *u*(168, 128).

of the spiral wave. In fact, the periodical forcing induces deformation of the media and the diffusion intensity thus is changed to force the spiral tip to the meander. An appropriate angle frequency of forcing can cause the tip of the spiral wave to move randomly, thus the center instability is generated. The distributions of frequency of the four sampled sites are also calculated and the spectra of power are also plotted in figures 3(*g*) and (*h*). Comparing the results in figures 3(*g*) and (*h*) with the results in figures 1(*e*) and (*f*), it is found that more prominent peaks are found and the maximal power is decreased. In the following section, we will discuss the differences of the power spectrum in different cases including the state of weak deformation, coexistence of the spiral wave and spiral turbulence and no evident deformation, etc. The angle frequency is increased and it is found that the spiral wave becomes shrunkun and/or stretched at $\omega = 0.3, 0.4, 0.5$.

As we know, the frequency of the rotating spiral wave in the Barkley model (which is similar to the MFHN in this paper) is mainly decided by the parameters a, b, ϵ [40, 41], and the parameter ε plays an important role in deciding the frequency of the spiral wave at fixed a and b. Therefore, it is interesting to investigate the transition of the spiral wave with the other different parameter ε. The transition of the spiral wave for $\varepsilon = 0.05$ with different angle frequencies and the results are plotted in figure 4.

The snapshots of figure 4 show that the spiral wave is surrounded by the spiral turbulence at $\varepsilon = 0.05$ and $\omega = 0.05$. The coexistence of the spiral wave and spiral turbulence is defined as that the spiral wave occupies the center of the media and it is always surrounded by the spiral turbulence. That is to say, the spiral wave in the center is not a fast enough traveling wave [42] to remove the spiral turbulence completely. Extensive studies confirm that the coexistence of the spiral wave and turbulence is still observed at $\varepsilon = 0.05$ and $\omega = 0.1$. The power spectra for the four time series of four sampled sites are also illustrated in figures 4(*e*) and (*f*) at the condition that the spiral wave coexists with the turbulence. The spiral wave loses stability within a few time units when the angle frequency is increased further; for example, the whole media finally become spiral turbulent and the spiral wave can be invaded by the spiral turbulence for $\omega = 0.2, 0.3, 0.4, 0.5$. Finally, we study the case for $\varepsilon = 0.06$

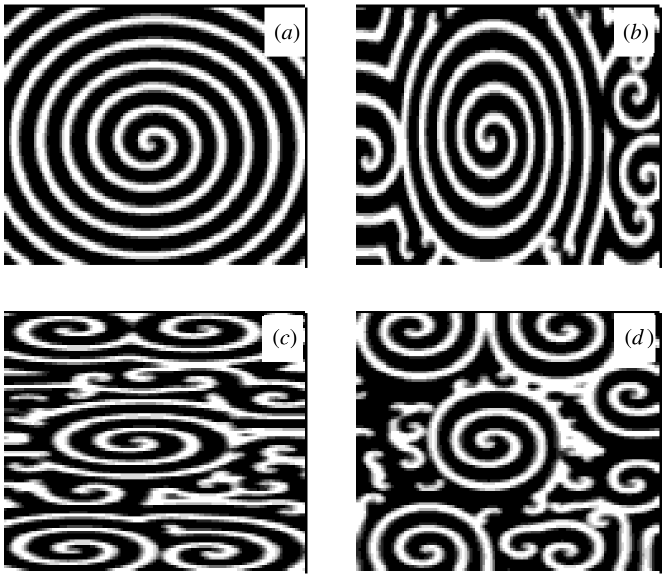

Figure 5. Evolution of the deformed spiral wave, $\varepsilon = 0.06$, $Af(t) = 0.4\cos 0.05t$, for $t = 200$ (*a*), $t = 600$ (*b*), $t = 1400$ (*c*) and $t = 1800$ (*d*) time units, and the snapshots are shown in gray.

and some results are shown in figure 5. The snapshots of figure 5 find that the spiral wave and spiral turbulence still can coexist with each other under the small angle frequency. However, the spiral wave encounters instability very soon and the whole media become turbulent when the angle frequency is increased further, for example, $\omega = 0.08, 0.1, 0.2, 0.3, 0.4, 0.5$. To our knowledge, the frequency of the spiral wave decreases with the increasing parameter ε; therefore, it becomes easy for the spiral wave to encounter instability with the increasing deformation frequency at a bigger parameter ε.

Above all, we have studied the instability of the spiral wave induced by the deformation of the media resulting from the periodical forcing by observing the snapshots of the activator. As we know, the noise can affect the dynamics of the excitable media and noise with enough intensity can induce the instability of the spiral wave [43]. The spatiotemporal noise is often defined as $\eta(x, y, t)$, and mean $\langle\eta(x, y, t)\rangle = 0$ and $\langle\eta(x, y, t)\rangle\langle\eta(x', y', t')\rangle = D_0\delta(x - x')\delta(y - y')\delta(t - t')$. The coefficient D_0 is the intensity of noise. We will discuss the case with weak intensity of noise and the numerical results are plotted in figure 6.

The snapshots of figure 6 confirm that no evident deformation of the spiral wave can be observed and the power spectrum for the time series of four sampled sites is illustrated in figures 6(*c*) and (*d*). Now it is important to estimate whether the instability of the spiral wave can be induced or not in a practical way though we can conclude the results by observing the snapshots of the activator. That is to say, we will discuss the difference of distribution of the frequency or spectrum in different cases, including the cases such as no evident deformation of the spiral wave as in figure 6, the weak deformation of the spiral wave as in figure 1, the coexistence of the spiral wave and turbulence as in figures 2, 4, 5, and the complete turbulence as in figure 3. In the case of no evident deformation of the spiral wave as in figure 6, the power spectrum marks the peak clear and prominent, and the peaks in the power spectrum are few and countable. In the case of weak deformation of the spiral wave as in figure 1, some new peaks are observed but the highest peak still dominates among all the frequencies. In the case

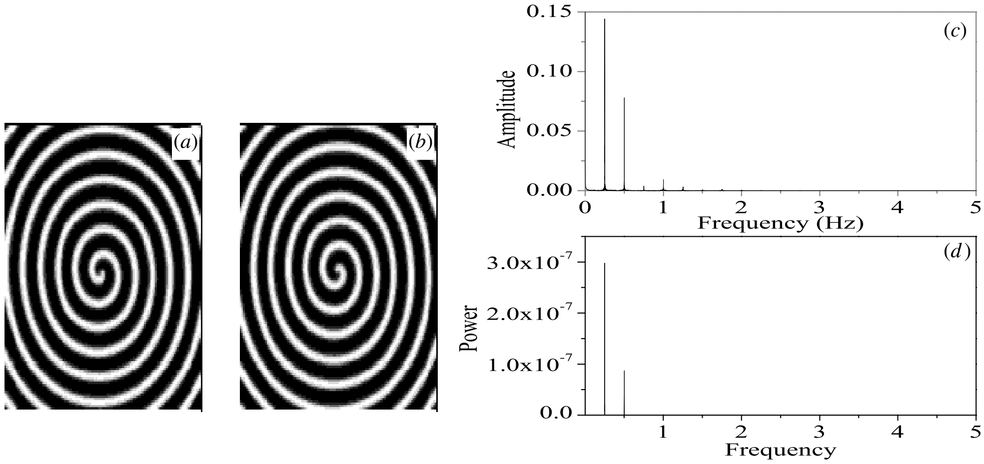

Figure 6. Distribution of frequency and evolution of the spiral wave when the weak deformation is induced by weak spatiotemporal noise, for $t = 200$ (*a*) and $t = 2000$ (*b*) time units. $\varepsilon = 0.045$, $Af(t) = \eta(x, y, t)$, the noise intensity $D_0 = 0.000\,06$ and for $t = 200$ (*a*), and $t = 2000$ (*b*) time units, and the snapshots are shown in gray. Power spectrum for the time series of mean activator of the four sampled sites $u(88, 128)$, $u(128, 88)$, $u(128, 168)$, $u(168, 128)$.

of the spiral wave coexisting with the spiral turbulence as in figure 4 (or 2, 5), we can find that more new peaks (frequencies) appear and the distribution of frequency is dense. Furthermore, in the case of complete instability of spiral wave as in figure 3, it is found that more peaks appear and the distribution of frequency is sparse. To the best of our knowledge, remarkable deformation of the spiral wave introduces local deformation in the media and more new spiral segments with new frequencies are induced, and it can step into turbulence finally as long as the deformation surpasses a threshold. For simplicity, we can detect and measure whether the instability of the spiral wave is induced or not by analyzing the power spectrum for time series of four sampled sites selected as in this paper, which lie in the media symmetrically along different directs. The results in figures 1(*e*), (*f*), 3(*g*), (*h*), 4(*e*), (*f*) and 6(*c*), (*d*) could be used to estimate whether the spiral wave encounters instability or not. In a word, analyzing the spectrum for the distribution of the frequency will tell us the truth we expect. Certainly, we never intend to deny the effective scheme by observing the snapshots to study the transition of the spiral wave directly.

4. Discussion and conclusion

In this paper, we investigate the transition of the spiral wave when periodical forcing-induced deformation is imposed on the whole media. Based on the nonlinear dynamics theory, the deformation effect on the spiral wave is simplified and simulated by adjusting the diffusion coefficients D_x and D_y with different periodical signals. It is different from the previous works which perturbs the controllable parameter with the same time-function synchronically, for example, the diffusion coefficients D_x and D_y are perturbed with the same signal. Our results focus on the elastic media, and the acreage is supposed to be invariable thus the shrinkage and stretch are compensatory. Our numerical results find that the transition of the spiral wave is decided by the ratio parameter ϵ of the media and the frequency of the external forcing deformation. It is interesting to find that the spiral wave can coexist with the spiral turbulence in an appropriate frequency of the external forcing deformation. We can conclude whether the spiral wave encounters instability or not by analyzing the distribution of the frequency of four sampled sites (activator) which often are selected symmetrically. According to the power spectrum for the time series of the four sampled sites, it is found that more new peaks appear and the distribution of the frequency becomes sparse as long as the spiral wave comes close to the critical condition for instability. Therefore, we suggest that we can detect four selected sites (selected symmetrically) and analyze the power spectrum to judge whether the spiral wave will encounter an instability. Comparing the results in this paper with the previous works, we extend the previous works and our studies are closer to describing the problem of instability of the spiral wave induced by the deformation in the media. The deformations along the lengthways and transverse ways are considered simultaneously, so that it can give some clue to studying the deformation of the spiral wave in cardiac tissue due to the shrinkage of tissue though we do not use the real cardiac tissue model.

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